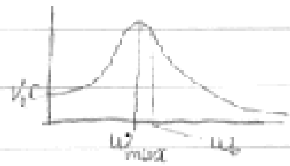
Uitwerking tentamen Golven en Optica 20/3/97 (zonder garantie)

1 a) $V_0 \cos\omega t = \frac{q}{C} + RI + L\frac{dI}{dt} = \frac{q}{C} + R\frac{dq}{dt} + L\frac{d^2q}{dt^2} \Rightarrow \frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = \frac{V_0}{L}\cos\omega t$

b) $z = Ae^{i(\omega t+\alpha)} \Leftrightarrow \left(-\omega^2 A + i\omega\frac{R}{L}A + \frac{1}{LC}A\right)e^{i(\omega t+\alpha)} = \frac{V_0}{L}e^{i\omega t}$ met $\omega_0^2 = \frac{1}{LC}$

$\Rightarrow (\omega_0^2 - \omega^2)A = \frac{V_0}{L}\cos\omega t$ en $\omega\frac{R}{L}A = \frac{V_0}{L}\sin\omega t$



mbv $\sin^2\omega t + \cos^2\omega t = 1 \longrightarrow A(\omega) = \frac{V_0/L}{\left[(\omega_0^2-\omega^2)^2 + \left(\frac{R}{L}\omega\right)^2\right]^{\frac{1}{2}}}$

$\frac{dA}{d\omega} = -\frac{1}{2}\frac{1}{[\cdots]^{3/2}}\left(2(\omega_0^2-\omega^2)\cdot 2\omega + \frac{2R^2}{L^2}\omega\right) = 0$

$\Rightarrow -2\omega_0^2 + 2\omega^2 + \frac{R^2}{L^2} = 0 \Rightarrow \omega_{max} = \sqrt{\omega_0^2 - \frac{R^2}{2L^2}} \rightarrow A_{max} = \frac{V_0/R}{[\omega_0^2 - R^2/4L^2]^{\frac{1}{2}}}$

c) $R = 0$, $A(\omega) = \frac{V_0/L}{\omega_0^2-\omega^2} \longrightarrow$ resonantie als $\omega = \omega_0 = \sqrt{\frac{1}{LC}}$

d) $\gamma = \frac{R}{L}$ $\quad Q = \frac{\omega_0}{\gamma} = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$ $\quad \omega_{max} = \omega_0\sqrt{1 - \frac{R^2}{2L^2\omega_0^2}} = \omega_0\sqrt{1-\frac{1}{2Q^2}}$

$A_{max} = \frac{V_0/(\omega_0 R)}{\sqrt{1 - \frac{R^2}{4L^2\omega_0^2}}} = V_0 C Q \frac{1}{\sqrt{1-\frac{1}{4Q^2}}}$ want $\frac{1}{\omega_0 R} = \frac{\sqrt{LC}}{R} = C\frac{L}{\sqrt{LC}R} = C\frac{\omega_0 L}{R} = C.Q$

Q is omgekeerd evenredig met relatieve breedte resonantiecurve $\left(Q \simeq \frac{\omega_0}{2\Delta\omega}\right)$; ook: voor onaangedreven systeem gaat amplitude naar $\frac{1}{e}\times$ voor ca. $\frac{Q}{\pi}$ perioden. Grote $Q \rightarrow$ geringe demping, scherpe resonantie

e) R dissipatie, C ~ potentiële energie als in veer, L zorgt voor traagheid, vgl. kinetische energie massa. Energie opgeslagen in magn. veld $L \Longleftrightarrow$ electr. veld C.

2. a) $m\frac{d^2x_1}{dt^2} = -kx_1 + k(x_2-x_1)$ $\qquad m\frac{d^2x_2}{dt^2} = -k(x_2-x_1)$

b) $x_1 = X_1\cos\omega t$, $x_2 = X_2\cos\omega t \rightarrow -m\omega^2X_1 = -kX_1 + k(X_2-X_1) \Rightarrow \frac{X_2}{X_1} = 2 - \frac{m}{k}\omega^2$ ①

$m\omega^2X_2 = -k(X_2-X_1) \Rightarrow \frac{X_2}{X_1} = \frac{1}{1-\frac{m}{k}\omega^2} \overset{①}{=} 2 - \frac{m}{k}\omega^2 \Rightarrow \omega^2 = \frac{k}{2m}(3\pm\sqrt{5})$

c) $\frac{X_2}{X_1} = 2 - \frac{m}{k}\omega^2 \Rightarrow \omega_1^2 = \frac{k}{2m}(3-\sqrt{5}) \longrightarrow \frac{X_2}{X_1} = \frac{1}{2} + \frac{1}{2}\sqrt{5}$ $\qquad (\omega_1^2 < \omega_2^2)$

$\omega_2^2 = \frac{k}{2m}(3+\sqrt{5}) \Rightarrow \frac{X_2}{X_1} = \frac{1}{2} - \frac{1}{2}\sqrt{5}$

d) Potentiële energie bovenste veer $\frac{1}{2}kx_1^2$, onderste $\frac{1}{2}k(x_2-x_1)^2$

$\Rightarrow$ verhouding energie $P = \frac{(X_2-X_1)^2}{X_1^2} = \left(\frac{X_2}{X_1} - 1\right)^2$

$\omega_1^2 \longrightarrow P_1 = \left(-\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^2 \simeq 0.382$; ω_2^2: $P_2 = \left(-\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^2 \simeq 2.618$

3 a) $f'_{min} = 40.10^6 + 400$ Hz $\quad f'_{max} = 40.10^6 + 2040$ Hz ; $\varphi = 0 \rightarrow$ geen doppler-shift $\rightarrow$ werkelijke $f = \frac{f_{min}+f_{max}}{2} = 40.10^6 + 1220$ Hz.

Alternatief: $f' = f_0/\left(1 - \frac{v_s \sin\varphi}{c}\right)$ vul in f'_{max} voor $\varphi = \frac{\pi}{2}$, f'_{min} voor $\varphi = -\frac{\pi}{2}$, etc.

b) $f'_{max} = 40.10^6 + 2040 = (40.10^6 + 1220)/\left(1 - \frac{v_s}{c}\right) \Rightarrow \frac{v_s}{c} = \frac{(40.10^6+2040) - (40.10^6+1220)}{40.10^6+2040}$

$v_s = \frac{820 \cdot 3.10^8}{40.10^6+2040} = 6.15$ km/s

4. a) $\gamma(\tau) = \langle E(t)E^*(t+\tau)\rangle / \langle|E|^2\rangle = \langle E(t)E^*(t+\tau)\rangle = \langle e^{i\omega\tau}e^{i[\phi(t)-\phi(t+\tau)]}\rangle =$

$= e^{i\omega\tau}\lim_{T\to\infty}\frac{1}{T}\int_0^T e^{i[\phi(t)-\phi(t+\tau)]}\,dt$

b) $\gamma(\tau) = \frac{e^{i\omega\tau}}{\tau_0}\int_0^{\tau_0} e^{i[\phi(t)-\phi(t+\tau)]}\,dt = \frac{e^{i\omega\tau}}{\tau_0}\int_0^{\tau_0-\tau} dt + \frac{e^{i\omega\tau}}{\tau_0}\int_{\tau_0-\tau}^{\tau_0} e^{i\delta}\,dt =$

$$= \left(\frac{\tau_0-\tau}{\tau_0} + \frac{\tau}{\tau_0}\, e^{i\Delta}\right) e^{i\omega\tau}$$

4b) Integreer over veel intervallen τ_0, met Δ random $\rightarrow \langle e^{i\Delta}\rangle = 0$

$\rightarrow \gamma(\tau) = \frac{\tau_0-\tau}{\tau_0}\, e^{i\omega\tau} \qquad |\gamma(\tau)| = \frac{\tau_0-\tau}{\tau_0} = \frac{1-0.25}{1} = 0.75 \rightarrow$

$\rightarrow$ partiële coherentie

4c) $\tau = 1.25 \rightarrow$ volledige incoherentie, geen contrast in het interferentiepatroon.

5a) $U_p = C \iint e^{ikr}\, dA = C \iint e^{ik(r_0+\delta r)}\, dA = C' \iint e^{ik\delta r}\, dA = C' \iint g(x,y)\, e^{ik\delta r}\, dx\, dy$

$\frac{\delta r_x}{x} \simeq \frac{X}{L} \qquad \delta r_x = \frac{X}{L} x$

$\delta r = \delta r_x + \delta r_y = \frac{X}{L} x + \frac{Y}{L} y$

$\rightarrow k\delta r = \frac{kX}{L} x + \frac{kY}{L} y = \mu x + \nu y$ met $\mu = \frac{kX}{L}$ en $\nu = \frac{kY}{L}$

b) $U = \int_{-a/2}^{a/2} dx \int_{-b/2}^{b/2} dy\, e^{i(\mu x+\nu y)} = \left[\frac{1}{i\mu} e^{i\mu x}\right]_{-a/2}^{a/2} \left[\frac{1}{i\nu} e^{i\nu y}\right]_{-b/2}^{b/2} = \frac{\sin(a\frac{\mu}{2})}{\mu/2} \cdot \frac{\sin(b\frac{\nu}{2})}{\nu/2}$

$I = |U|^2 = \frac{\sin^2(a\frac{\mu}{2})}{(\mu/2)^2} \frac{\sin^2(b\frac{\nu}{2})}{(\nu/2)^2}$ met $\mu = \frac{k}{L} X = \frac{2\pi}{\lambda L} X \qquad \nu = \frac{2\pi}{\lambda L} Y$

c) $U = U_x U_y \qquad U_x = \int_{-a/2}^{a/2} \cos(\pi \frac{x}{a})\, e^{i\mu x}\, dx = \int_{-a/2}^{a/2} \frac{e^{i(\pi\frac{x}{a}+\mu x)} + e^{i(-\pi\frac{x}{a}+\mu x)}}{2}\, dx =$

$= \frac{\sin(a\frac{\mu}{2}+\frac{\pi}{2})}{\frac{1}{2}(\mu+\frac{\pi}{a})} + \frac{\sin(a\frac{\mu}{2}-\frac{\pi}{2})}{\frac{1}{2}(\mu-\frac{\pi}{a})} = \cos(a\frac{\mu}{2})\left[\frac{1}{\frac{1}{2}(\mu+\frac{\pi}{a})} - \frac{1}{\frac{1}{2}(\mu-\frac{\pi}{a})}\right]$

$U_y = \int_{-b/2}^{b/2} \left(0.5 + 0.5\cos\left(\frac{2\pi y}{b}\right)\right) e^{i\nu y}\, dy = \sin(b\frac{\nu}{2})\left[\frac{1}{\nu} - \frac{1}{\nu+\frac{2\pi}{b}} - \frac{1}{\nu-\frac{2\pi}{b}}\right]$

d) Zijlobben kleiner, minder verwarrend bij dubbelsterren waarvan de een veel lichtzwakker dan de ander.